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Logarithmic corrections to the FRW brane cosmology from 5d Schwarzschild-deSitter black hole

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ABSTRACT

Thermodynamics of 5d SdS black hole is considered. Thermal fluctuations define the (sub-dominant) logarithmic corrections to black hole entropy and then to Cardy-Verlinde formula and to FRW brane cosmology. We demonstrate that logarithmic terms (which play the role of effective cosmological constant) change the behavior of 4d spherical brane in dS, SdS or Nariai bulk. In particular, bounce Universe occurs or 4d dS brane expands to its maximum and then shrinks. The entropy bounds are also modified by next-to-leading terms. Out of braneworld context the logarithmic terms may suggest slight modification of standard FRW cosmology.

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1 Introduction

The deSitter space always attracts much attention in cosmology and gravity. This is caused by several reasons. First of all, according to the theory of inflationary Universe the very early Universe eventually has passed deSitter (dS) phase. Second, recent astrophysical data indicate that modern Universe is (or will be in future) also in deSitter phase. Third, dS is very attractive from the theoretical point of view due to its highly symmetric nature (like flat space). This is also the reason why dS space was frequently considered as candidate for ground state in quantum gravity.

According to recent studies the dS quantum gravity should be quite unusual theory in many respects [1]. In connection with braneworld scenario [2] it is expected that there occurs dS/CFT correspondence [3, 4]. In one of its versions, dS/CFT correspondence indicates that properties of 5d classical dS space are related with those of dual CFT living on the four-dimensional boundary (which may be also dS). Despite the fact that explicit examples of such consistent dual CFTs are not constructed yet, one can still get a lot of information from dS/CFT correspondence. In particular, starting from five-dimensional Schwarzschild-deSitter (SdS) black hole (which should be relevant to the description of 4d dual CFTs at non-zero temperature) one can easily get the Friedmann-Robertson-Walker (FRW) brane cosmology. The corresponding FRW brane equation may be often written in so-called Cardy-Verlinde (CV) form [5]. There was much activity recently (see [6, 7]) in the study of FRW brane cosmology in CV form when bulk space is dS or SdS space and in the corresponding investigations of thermodynamical properties of dS black holes.

In the present paper we consider 5d SdS black hole and calculate the corresponding thermodynamical quantities. Taking into account thermal fluctuations defines the logarithmic corrections to both cosmological and black hole entropies. As a result the CV formula and FRW brane cosmology receive the (sub-dominant) logarithmic corrections, in the way similar to FRW brane cosmology in AdS black hole bulk [8]. It is interesting that such sub-dominant terms slightly change the entropy bounds appearing in CV formulation.

The paper is organized as follows. In the next section we find entropy, free energy and thermodynamical energy for SdS black hole which is the space with two horizons and for Nariai black hole (when both horizons coincide). Using the logarithmic corrections to the entropy the corrected CV formula is established. The corresponding FRW brane equation with logarithmic terms is found. Section three is devoted to the qualitative study of FRW brane cosmology where next-to-leading (logarithmic) terms play the role of small effective cosmological constant. It is explicitly demonstrated that 4d spherical brane behaves in a different way when log-terms are present. In particular, bounce Universe may occur or dS brane reaches its maximum and then shrinks. Its behavior depends also from the choice of bulk: dS, SdS or Nariai space. In section four we consider standard 4d FRW cosmology and show that even in this case, due to log-corrections to four-dimensional cosmological entropy the FRW equa-

tion and CV formula may get modified. The correction terms may be interpreted as dust. Some summary and outlook are given in the last section. General derivation of logarithmic corrections to entropy (due to thermal fluctuations) is presented in the Appendix A. Penrose diagram of SdS space is drawn in Appendix B.

2 Logarithmic Corrections to Cardy-Verlinde formula and FRW brane cosmology in SdS bulk

Let us consider the thermodynamics of SdS black hole in five dimensions. The SdS black hole is a constant curvature solution of the Einstein equation, which follows from the action

$$S = \int d^5x \sqrt{-\hat{G}} \left\{ 116\pi G_5 \hat{R} + \Lambda \right\} . \quad (1)$$

Here \hat{R} is the scalar curvature, Λ is the (positive) cosmological constant and G_5 denotes the five-dimensional Newton constant.

The metric of the five-dimensional SdS black hole is given by

$$\begin{aligned} ds_5^2 &= \hat{G}_{\mu\nu} dx^\mu dx^\nu \\ &= -e^{2\rho} dt^2 + e^{-2\rho} da^2 + a^2 \sum_{ij}^3 g_{ij} dx^i dx^j , \\ e^{2\rho} &= 1a^2 (-\mu + k2a^2 - a^4 l^2) , \end{aligned} \quad (2)$$

where l represents the curvature radius of the SdS bulk space and is related with the cosmological constant $\Lambda = 12l^2$. The three-dimensional metric, g_{ij} , is the metric of an Einstein space with Ricci tensor given by $r_{ij} = kg_{ij}$, where the constant $k = \{-2, 0, +2\}$ in our notations. The metric of the hypersurface with constant a is then negatively-curved, spatially flat, or positively-curved depending on the sign of k . For SdS background, only $k = 2$ is the static solution, but when $k = -2, 0$, the coordinate a plays a role of the (second) time coordinate and the solutions are time-evolving and cosmological ones with a big-bang singularity at $a = 0$.

The mass of the black hole is parametrized by a constant μ , and μ can be expressed in terms of the horizon radius a_H :

$$\mu = a_H^2 (-a_H^2 l^2 + k2) . \quad (3)$$

The horizon radius a_H is the solution of the equation $\exp[2\rho(a_H)] = 0$, which corresponds to (3)

$$a_H^2 = kl^2 4 \pm 12 \sqrt{k^2 l^4 4 - 4\mu l^2} \quad (4)$$

Note that, when $k = 2$ SdS black hole has two horizons a_H , that corresponds to the upper and lower signs in Eq.(4) (the cosmological and black hole horizons, respectively). Hereafter we denote black hole horizon by a_{HB} and cosmological one by a_{HC} . When $k = 0, -2$, there is no horizon since the right-hand side in (4) becomes imaginary or negative for positive μ . Then in the following we consider mainly $k = 2$ case.

One can define two Hawking temperatures corresponding to the two horizons:

$$T_H = \left| 14\pi de^{2\rho} da \Big|_{a=a_{BH}, a_{CH}} \right| = \begin{cases} 12\pi a_{BH} - a_{BH}\pi l^2 & \text{for the black hole horizon} \\ -12\pi a_{CH} + a_{CH}\pi l^2 & \text{for the cosmological horizon} \end{cases} . \quad (5)$$

The Cardy-Verlinde (CV) formula [5] (see also [9]) is derived from the thermodynamical properties of the five-dimensional black hole. So let us summarize the calculation of the thermodynamical quantities like the free energy F , the entropy \mathcal{S} , and the energy E by following the method in [10]. After Wick-rotating the time variable $t \rightarrow i\tau$, the free energy F can be obtained from the action eq.(1) as $F = -TS$. The classical solutions for \hat{R} and Λ are given by $\hat{R} = 20l^2$ and $\Lambda = -1216\pi G_5 l^2$. Then the classical action (1) takes the form

$$\begin{aligned} S &= 816\pi G_5 l^2 \int d^5x \sqrt{-\hat{G}} , \\ &= W_3 T 816\pi G_5 l^2 \int_{a_H}^{\infty} da a^3 , \end{aligned} \quad (6)$$

where W_3 is the volume of the unit three-sphere and τ has a period of $1T$. The expression for S contains the divergence coming from large a . In order to subtract the divergence, we regularize S (6) by cutting off the integral at a large radius a_{\max} and subtracting the solution with $\mu = 0$ in the same way as in [10]:

$$S = W_3 T 816\pi G_5 l^2 \left\{ \int_{a_H}^{a_{\max}} da a^3 - e^{\rho(a_{\max}) - \rho(a_{\max}; \mu=0)} \int_0^{a_{\max}} da a^3 \right\} . \quad (7)$$

The factor $e^{\rho(a_{\max}) - \rho(a_{\max}; \mu=0)}$ is chosen so that the proper length of the circle which corresponds to the period $1T$ in the Euclidean time at $a = a_{\max}$ coincides with each other in the two solutions. Taking $a_{\max} \rightarrow \infty$, one finds

In this paper, we are interested primarily in the corrections to the entropy (??) that arise due to thermal fluctuations. The leading-order correction has been found for a generic thermodynamic system [11]. The entropy is calculated in terms of a grand canonical ensemble, where the corresponding density of states, ρ , is determined by performing an inverse Laplace transformation of the partition function⁴. The

⁴The reader is referred to Refs. [11, 12] for details and related discussion in refs.[13]

integral that arises in this procedure is then evaluated in an appropriate saddle-point approximation. The correction to the entropy follows by assuming that the scale, ϵ , defined such that $\mathcal{S} \equiv \ln(\epsilon\rho)$, varies in direct proportion to the temperature, since this latter parameter is the only parameter that provides a physical measure of scale in the canonical ensemble. The final result is then of the form [12]:

$$\mathcal{S} = \mathcal{S}_0 - \frac{1}{2} \ln C_v + \dots, \quad (8)$$

where C_v is the specific heat of the system evaluated at constant volume and \mathcal{S}_0 represents the uncorrected entropy. The derivation of (8) is given in Appendix A. In the case of the SdS black hole, the entropy is given by Eq. (??). The specific heat of the black hole is determined in terms of this entropy:

$$C_v \equiv dEdT_H = 32a_H^2 - l^2 2a_H^2 + l^2 \mathcal{S}_0. \quad (9)$$

The above expression (9) is valid for both of the black hole and cosmological cases. For consistency, the condition $a_H^2 > l^2/2$ should be satisfied to ensure that the specific heat is positive. In the limit $a_H^2 \gg l^2/2$, $C_v \approx 3\mathcal{S}_0$, and this implies that [12]

$$\mathcal{S} = \mathcal{S}_0 - 12 \ln \mathcal{S}_0 + \dots. \quad (10)$$

Using the form of the logarithmic correction (10) to the entropy, it is now possible to derive the corresponding corrections to CV formula. We begin by recalling that the four-dimensional energy, which can be derived from the FRW equation of motion for a brane propagating in an SdS bulk is given by

$$E_4 = \pm 3W_3 l \mu 16\pi G_5 a \quad (11)$$

(+ corresponds to the black hole horizon and $-$ to the cosmological one) and is related to the five-dimensional energy (??) of the bulk black hole such that $E_4 = (l/a)E$ [14]. This implies that the temperature T , associated with the brane should differ from the Hawking temperature (5) by a similar factor [14]:

In determining the corrections to the entropy, a crucial physical quantity is the Casimir energy E_C [5], defined in terms of the four-dimensional energy E_4 , pressure p , volume $W = a^3 W_3$, temperature T and entropy \mathcal{S} :

Moreover, in the limit where the logarithmic correction in Eq. (??) is small, it can be shown, after substitution of Eqs. (??), (11) and (??), that the four-dimensional and Casimir energies are related to the uncorrected entropy by [15]

If we consider the brane universe in the SdS black hole background, the four-dimensional FRW equation, which describes the motion of the brane universe, also receives corrections as a direct consequence of the logarithmic correction arising in Eq.(??). In general, the Hubble parameter H is related with the four-dimensional (Hubble) entropy (which is identified with bulk black hole entropy, see corresponding proof for AdS bulk in ref. [14] and for dS bulk-in [7])

$$H^2 = (2G_4 W)^2 \mathcal{S}^2, \quad (12)$$

Here the effective four-dimensional Newton constant G_4 is related to the five-dimensional Newton constant G_5 by $G_4 = 2G_5/l$. The formula (12) is correct in both cases: when the brane crosses black hole horizon ($a = a_{BH}$) and when the brane crosses the cosmological one ($a = a_{CH}$). As we will see in the next section, we extend the equation corresponding to (12) to the case for $a \neq a_{BH}, a_{CH}$. The extended equation (18) coincides with (12) in both cases $a = a_{BH}$ and $a = a_{CH}$. By substituting Eq. (??) into Eq. (12), it can be shown by employing Eqs. (3), (??), (??), (??), (??) and (??) that the four-dimensional FRW equation is

$$\begin{aligned} H^2 &= (2G_4 W)^2 \left[\left(4\pi a 3\sqrt{2} \right)^2 |E_C(E_4 - 12E_C)| - 4\pi a 3\sqrt{2} E_4 (4E_4 - 3E_C) (2E_4 - E_C) E_C \right. \\ &\quad \times \left. \sqrt{|E_C(E_4 - 12E_C)|} \ln \left(4\pi a 3\sqrt{2} \sqrt{|E_C(E_4 - 12E_C)|} \right) \right] \\ &= 1a_H^2 - 8\pi G_4 3\rho - 2G_4 W l \ln \mathcal{S}_0, \end{aligned} \quad (13)$$

Here the logarithmic corrections have been included up to first-order in the logarithmic term, the effective energy density is defined by $\rho = |E_4|/W$ and $W = a_H^3 W_3$ parametrizes the spatial volume of the world-volume of the brane. In the limit where the scale factor a of the brane coincides with the horizon radius a_H of the black hole, the first and second terms on the right-hand-side of Eq. (13) are identical to the FRW equation for the *space-like* brane in SdS background whose signs of the terms are the opposite to SAdS background [7, 15]. It is interesting that even if we did not assume the space-like brane, the Hubble equation (13) agrees with the case for space-like brane which is the brane for the Wick-rotated version of standard FRW equation.

Then the logarithmic corrections for the FRW equation are given by the third term on the right-hand side in terms of the uncorrected entropy (??) of the black hole.

In the usual four-dimensional cosmology, the (first) FRW equation is given by

$$\begin{aligned} H^2 &= 8\pi G 3\rho - 1a^2, \\ \rho &= \rho_m + \Lambda 8\pi G. \end{aligned} \quad (14)$$

Here Λ is a cosmological constant and ρ_m corresponds to the energy density of the matter. Typically in case that the matter is radiation, ρ is proportional to $1a^4$. Then

Eq.(13) tells that, if we neglect the logarithmic correction, the obtained energy density corresponds to the radiation and the cosmological constant should vanish. On the other hand, by comparing (13) and (14), the logarithmic correction can be regarded as a small effective cosmological constant by identifying

Particularly, we consider the Nariai black hole which is the most simple case. In this case, the second term of (4) is zero, namely $a_H^2 = l^2 2$. Then black hole horizon coincides with cosmological horizon.

The Hubble equation (13) for this case looks

$$H^2 = 1l^2 - 16G_4 2^{3/2} W_3 l^4 \ln \mathcal{S}_0 , \quad (15)$$

$$\mathcal{S}_0 = W_3 2^{3/2} l^2 16G_4 . \quad (16)$$

In the Nariai limit where $\mu = l^2 4$, the expression (5) for the Hawking temperature seems to vanish but this might not be true since $e^{2\rho} = 0$ in the region between the black hole and cosmological horizons, which tells that the coordinates t and a are degenerate or ill-defined in the region. Since the SdS solution is not asymptotically flat, there is an ambiguity to rescale the time coordinate by a constant factor. We now introduce the following new coordinates \tilde{a} and \tilde{t} :

3 Qualitative Dynamics of the Brane Cosmology

In this section, we investigate the asymptotic behavior of the FRW brane cosmology when the logarithmic corrections to the CV formula are included. Formally, the FRW equation (13) holds precisely at the instant when the brane crosses black hole and cosmological horizons. Here we extend the analysis to consider an arbitrary scale factor a where the world-volume of the brane is given by the line-element $ds_4^2 = d\tau^2 + a^2(\tau)g_{ij}dx^i dx^j$. Thus, around each horizon we assume the FRW equation as follows:

$$H^2 = 1a^2 - 8\pi G_4 3\rho - 2G_4 W l \ln \mathcal{S}_0 , \quad (17)$$

where $W = a^3 W_3$ and $\mathcal{S}_0 = W_3 a^3 / (4G_5)$. This equation differs by several signs from the corresponding FRW brane equation with log-corrections obtained in ref.[8] where bulk is AdS black hole. On the black hole and cosmological horizon eq.(17) agrees with eq.(13). We also extend the result of the previous section to the general k :

$$H^2 = k2a^2 - 8\pi G_4 3\rho - 2G_4 W l \ln \mathcal{S}_0 , \quad (18)$$

Eq.(18) can be rewritten in such a way that it represents the conservation of energy of a point particle moving in a one-dimensional effective potential, $V(a)$:

$$(dad\tau)^2 = k^2 - V(a) \quad (19)$$

$$\begin{aligned} V(a) &\equiv 8\pi G_4 3a^2 \rho + 2G_4 a^2 W l \ln \mathcal{S}_0 \\ &= \mu a^2 + 2G_4 W_3 l a \ln (W_3 a^3 2l G_4) , \end{aligned} \quad (20)$$

where, in this interpretation, the variable a represents the position of the particle. Since $\rho \propto a^{-4}$, the first term in the effective potential (20) redshifts as a^{-2} as the brane moves away from the black hole horizon. This term is often referred to as the ‘dark radiation’ term.

To proceed in the analogy with [8], let us briefly recall the behavior of the standard FRW cosmology, whose effective potential includes only the first term on the right-hand side of Eq. (20). The behavior of this potential is illustrated in Figure 1. The brane exists in the regions where the line $k/2 \geq V(a)$ (so that $H^2 > 0$). Then, we only have the case of $k = 2$ which is the spherical brane. The spherical brane starts from $a = \infty$ and reaches its minimum size at $a = a_{\min} = \sqrt{\mu}$ and then it re-expands.

From eq.(11) the energy density $\rho = E_4/W$ looks like $3\mu 8\pi G_4 a^4$, then the first term in eq.(20) is rewritten as μ/a^2 :

$$V(a) \equiv \mu a^2 + 2G_4 W_3 l a \ln \mathcal{S}_0 . \quad (21)$$

For the Nariai black hole, the mass μ takes the particular value $\mu = k^2 l^2 16$ which is the largest mass for the SdS black hole, since the inside of square root in eq.(4) must be positive. Then the behavior of the potential for Nariai BH is bigger than in SdS case as illustrated by thin line in Figure 2 so that its minimum size a_{\min} is bigger than the minimum size of SdS.

Next, one considers the behavior of the effective potential with logarithmic corrections. From eq.(21), there are several cases which depend on the parameters G_4, W_3, l, μ . If the coefficient $2G_4 W_3 l$ of second term in eq.(21) is equal to or less than $\sqrt{\mu}$, the behavior of the potential is not so changed from Figure 1. But when the coefficient $2G_4 W_3 l$ of the second term in eq.(21) is large compared with $\sqrt{\mu}$, the behavior of the potential changes from thin line to thick line as illustrated in Figure 2.

For Nariai black hole, the ratio of μ and the second term in eq.(21) is smaller than that of SdS case since μ is always bigger than the mass of SdS black hole. Then the behavior of the effective potential is similar to Figure 2 but smaller than that of SdS case.

As an explicit example one can take five-dimensional deSitter background instead of SdS background. The dS metric is given by

$$\begin{aligned} ds_5^2 &= -e^{2\rho} dt^2 + e^{-2\rho} da^2 + a^2 \sum_{ij}^3 g_{ij} dx^i dx^j , \\ e^{2\rho} &= 1 - a^2 l^2 , \end{aligned} \quad (22)$$

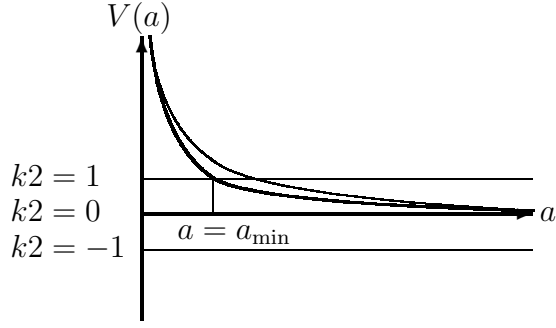


Figure 1: The effective potential for the FRW brane Universe in SdS bulk. For $k = 2$, the spherical brane starts at $a = \infty$ and reaches its minimum size at $a = a_{\min}$ and then it re-expands. For Nariai black hole, the effective potential is larger than that in the other SdS cases as illustrated by thin line.

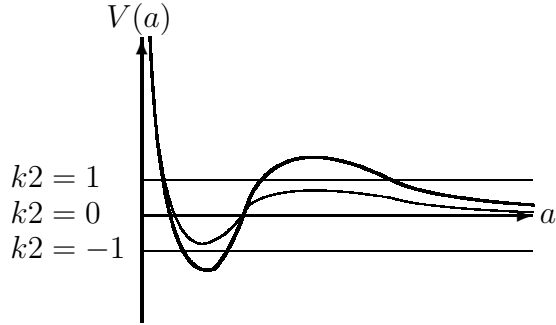


Figure 2: The effective potential for the FRW Universe in SdS bulk when logarithmic corrections are included. There are several cases which depend on the parameters G_4, W_3, l .

which is the massless case $\mu = 0$, $k = 2$ in eq.(22). Then, horizon radius looks like $a_H = l$. From eq.(13), the FRW equation for dS case also takes simple form as

$$H^2 = 1a_H^2 - 2G_4 W l \ln \mathcal{S}_0 , \quad (23)$$

where $\mathcal{S}_0 = W_3 a_H^3 4G_5$.

One can extend the FRW equation to general a and k :

$$H^2 = k2a^2 - 2G_4 W l \ln \mathcal{S}_0 . \quad (24)$$

Here $\mathcal{S}_0 = W_3 a^3 4G_5$, $W = a^3 W_3$, again. This equation defines the effective potential $V(a)$ as

$$V(a) = 2G_4 a W_3 l \ln \mathcal{S}_0 . \quad (25)$$

The behavior for the effective potential for dS bulk is illustrated in Figure 3. When $k = 0, -2$, the brane starts from $a = 0$ and reaches its maximal size a_{\max} and then it re-collapses. Note that the behavior of the effective potential with logarithmic correction for FRW universe in deSitter bulk differs from the one in SdS bulk (Figure 2). If the maximum of the effective potential V_{\max} is larger than 1, there are two solutions for $k = 2$ case. In one case, the brane started at $a = 0$ reaches its maximum and shrinks. In another case, the brane started at $a = \infty$ shrinks and reaches its minimum and reexpands, which is the bounce universe case.

When there are no logarithmic corrections, Eq.(24) has a simple form:

Thus, we demonstrated that the evolution of spherical brane which could correspond to our observable early Universe depends explicitly from the choice of bulk (dS, SdS or Nariai space) and from the inclusion (or not) of log-corrections.

4 Logarithmic corrections for four-dimensional FRW cosmology

In this section, we forget for the moment about the braneworld and discuss the role of logarithmic corrections to usual 4d cosmology and to 4d CV formula (see [5] and for CV formula in 4d dS space, see[16]). One starts from the Einstein gravity with positive cosmological constant $\Lambda_4 > 0$. Then the standard FRW equation has the following form:

If we assume $E = \delta E$ (in the absence of matter), by using (??) one obtains an expression for the Casimir energy:

Thus, we found that logarithmic correction to the entropy may lead to inducing of small effective cosmological constant in FRW equation. Eventually, this may have some cosmological applications.

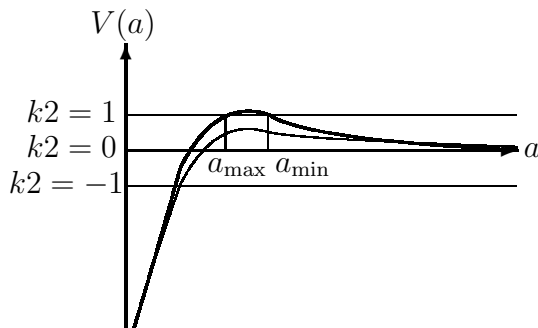


Figure 3: The behavior of the effective potential for FRW Universe in deSitter bulk with logarithmic corrections. There are two types of behavior for spherical brane $k = 2$. For the case of thick line, the brane starts from $a = 0$ and reaches its maximal size $a = a_{\max}$ and then it re-collapses, or the brane starts from $a = \infty$ and reaches its minimum size at $a = a_{\min}$ then it re-expands. For the case of thin line, the brane starts from $a = 0$ and expands to infinity.

5 Discussion

In the present paper we discussed the role of logarithmic corrections which appear in SdS black hole entropy to FRW brane cosmology. The relation between black hole entropy and Hubble parameter is controlled by dS/CFT correspondence. These, next-to-leading corrections in FRW equation may be interpreted as small effective cosmological constant which qualitatively changes the evolution of spherical brane. The examples of the spherical brane evolution are presented without (or with) logarithmic terms and for different bulk: dS, SdS or Nariai space. Eventually, if our brane FRW Universe is embedded into SdS bulk (or AdS black hole [8]), these next-to-leading terms may be very important in cosmology as we explicitly demonstrated.

Let us now comment their role in the entropy bounds estimations. If we define the four-dimensional Hubble, Bekenstein-Hawking and Bekenstein entropies by [5]

Finally, one can note that similar considerations are applied in the study of FRW or anisotropic brane cosmology with logarithmic corrections for another types of dS bulk black holes.

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Appendix

A Logarithmic corrections to the entropy

In this section, we review briefly the calculation of the log correction to the entropy. First, let us recall the expression for the partition function in the grand canonical ensemble by

$$Z(\beta) = \int e^{-\beta E} \rho(E) dE . \quad (26)$$

Here $\rho(E)$ is the density of states, β is the inverse temperature, $\beta = 1/T$, and we set $k_B = 1$, so that the temperature has the dimension of energy. From the classical thermodynamical relation between free energy F , energy E and entropy S :

$$F = E - TS , \quad F = -T \ln Z , \quad (27)$$

the partition function eq.(26) can be written as follows:

$$e^{-\beta F} = \int dE e^{-\beta E + S(E)} , \quad (28)$$

where $\rho(E) = e^{S(E)}$.

The function $-\beta E + S(E)$ can be expanded around the energy of thermal equilibrium point E_0 as

For Schwarzschild-deSitter spacetime, there are two kinds of temperatures corresponding to two horizons, black hole horizon and cosmological one. The system is not thermodynamically stable. However, the system should be adiabatic since one can define the temperature in the vicinity of each horizon. Furthermore, future black hole and cosmological horizons are separated from each other as we will see in the next Appendix B. Then we may discuss the thermodynamics near the horizon.

B Penrose diagram for Schwarzschild-deSitter black hole.

The Penrose diagram for Schwarzschild-deSitter black hole is given in Figure 4. We can find the future black hole horizon is causally separated from the cosmological

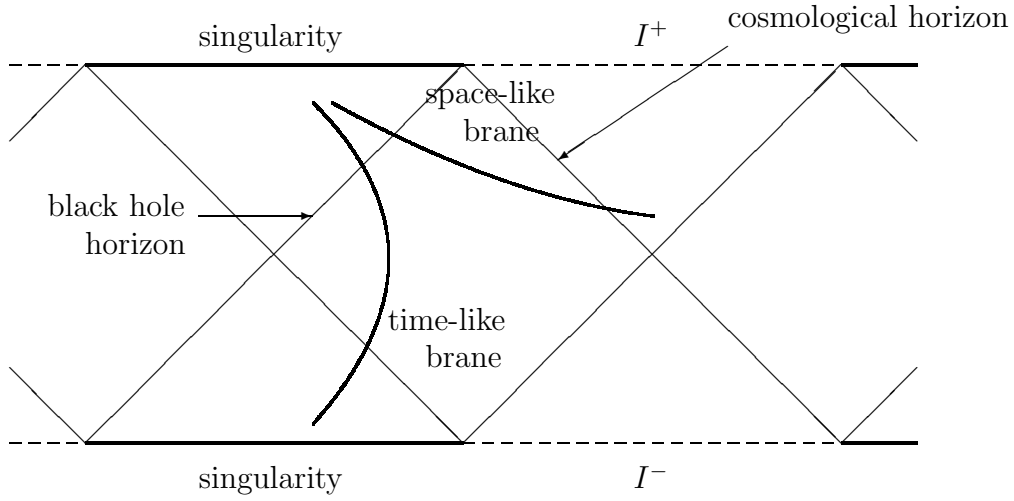


Figure 4: The time-like and space-like branes in the Penrose diagram of the Schwarzschild-deSitter spacetime.

one. Then any particle in a region between the black hole and cosmological horizons will cross one and only one of the future horizons. Then such a particle observes the energy (the entropy, etc.) associated with the horizon that the particle crosses.

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